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A FUNCTIONAL STRESS INTENSITY APPROACH TO MULTIPLY CRACKED, PARTIALLY AUTOFRETTAGED CYLINDERS

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The functional stress intensity approach is presented for a partially autofrettaged, thick-walled cylinder. This approach is a combination of a series of methods developed for the computation of stress intensity factors for multiple-radial cracks emanating from the inner or the outer surface of a hollow cylinder. The numerical method is mainly based on the finite element method using 12-node quadrilateral, isoparametric elements with singular
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elements around a crack tip. The difficulty due to the presence of initial stresses in the finite element method is obviated by the method of thermal simulation which replaces the residual stresses existing in an autofrettaged cylinder by an active thermal load. The weight function method is incorporated to reduce the repeated computations of stress intensity factors of the same geometrical configuration subjected to various external loads and residual stresses. The functional stress intensity factor is introduced to overcome the difficulty in seeking the weight function itself.

Numerical results of functional stress intensity factors are given for multiple cracks radiating from the bore or from the outer surface of a cylinder having an external diameter twice that of an internal diameter. A linear superposition of these results gives the resultant stress intensity factor of a cracked geometry subjected to combined external loads and initial stresses. It is highly possible to extend the method outlined in this report for elastic-perfectly plastic material to strain-hardening materials.

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INTRODUCTION

An analytic method is not available for the computation of stress intensity factors for multiple-radial cracks in a thick-walled cylinder. The computation must depend on various numerical methods.[1-5] Due to increasingly successful applications of the finite element technique in structural analysis, the author decided to use higher order finite elements with the aid of special crack-tip elements to study multiply cracked cylinders. Similar to the quarter-point element in an 8-node quadrilateral element,[6,7] a special crack-tip element was developed[8] for a 12-node quadrilateral, isoparametric element. Both the 8-node and 12-node quadrilaterals have been implemented in the popular finite element computer code NASTRAN.[8,9] The dummy user element facility of NASTRAN is used for the implementation. Another finite element computer code APES,[10] which was written specifically for the use of 12-node quadrilateral, isoparametric elements, has also been used for fracture analysis of cracked hollow cylinders. Quite accurate results of stress intensity factors have been obtained using either NASTRAN or APES for multiple-radial cracks emanating from the bore of a tube.[11] These results are in good agreement with results reported by Tracy[12] using the method of modified mapping collocation.

To increase the maximum internal pressure a cylinder can contain elastically and retard the growth of radial cracks near the bore, it is a common practice to introduce compressive residual stress near the bore by an autofrettage process. However, the residual stress in the cylinder causes

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References are listed at the end of this report.

increased difficulties in the estimation of stress intensity factors. One of the difficulties is that results of residual stress distribution vary based on different assumptions by different investigators. Another difficulty is the lack of an initial stress analysis capability in NASTRAN and APES. A method is developed in Reference 13 so that NASTRAN or APES can be used for the computation of stress intensity factors for cracked cylinders with residual stress distribution given in the closed form expressions.[14] The finite element results[15] are in close agreement with Parker's results[16] using modified mapping collocation.

While the autofrettage process produces favorable compressive residual stress near the bore, it also yields a tensile residual stress near the outer cylindrical surface. The sum of this stress and the tensile stress due to a bore pressure may be high enough to cause crack initiation and propagation from the outer surface of the cylinder. This requires the computation of stress intensity factors for externally cracked cylinders. Several investigators have reported their results on this subject.[12,17-19]

In order to reduce repeated finite element computations, the weight function method[20] is used together with the finite element method. In this report the functional stress intensity factor approach is summarized for both internally cracked and externally cracked, partially autofrettaged, pressurized thick-walled cylinders.

RESIDUAL STRESSES AND THERMAL SIMULATION

The residual stress distribution in an autofrettaged thick-walled cylinder has been studied by a large number of investigators. There is considerable disagreement in their results due to different assumptions which must be made in order to make the problem mathematically tractable. Detailed discussions of the results and the associated assumptions are given in References 14 and 21. Under the combination of assumptions that the material is incompressible, elastic-perfectly plastic, and obeys the von Mises' yield criterion, and that the cylinder is under the condition of plane strain, the following closed form solution for residual stresses is obtained for an elastically unloaded cylinder after partial autofrettage:

$$\sigma_r(r) = \begin{cases} \frac{\sigma_o}{\sqrt{3}}\{2 \log \frac{r}{\rho} - 1 + \frac{\rho^2}{b^2} - P_1(\frac{1}{b^2} - \frac{1}{r^2})\} & 1 \leq r \leq \rho \quad (1) \\ \frac{\sigma_o}{\sqrt{3}} (\rho^2 - P_1)(\frac{1}{b^2} - \frac{1}{r^2}) & \rho \leq r \leq b \quad (2) \end{cases}$$

$$\sigma_\theta(r) = \begin{cases} \frac{\sigma_o}{\sqrt{3}}\{2 \log \frac{r}{\rho} + 1 + \frac{\rho^2}{b^2} - P_1(\frac{1}{b^2} + \frac{1}{r^2})\} & 1 \leq r \leq \rho \quad (3) \\ \frac{\sigma_o}{\sqrt{3}} (\rho^2 - P_1)(\frac{1}{b^2} + \frac{1}{r^2}) & \rho \leq r \leq b \quad (4) \end{cases}$$

where bore radius is taken as unit length, b is the outer radius of the hollow cylinder, ρ is the radius of the elastic-plastic interface during pressurization, σ_o is the uniaxial yield stress in tension and compression, and

$$P_1 = P_1(\rho) = \frac{b^2}{b^2 - 1} (1 - \frac{\rho^2}{b^2} + 2 \log \rho) \quad (5)$$

This residual stress distribution is used in this report as a basis to develop a method to compute stress intensity factors for cracks in such a stress field. It has been shown in Reference 13 that the thermal stresses in the cylinder subjected to a thermal load

$$T(r) = \begin{cases} T_o - \dfrac{(T_o - T_\rho)}{\log \rho} \log r & 1 \leq r \leq \rho \\ T_\rho & \rho \leq r \leq b \end{cases} \tag{6}$$

are equivalent to the residual stresses in Eqs. (1) through (4) if the temperature gradient and the yield stress satisfy

$$\frac{E\alpha(T_o - T_\rho)}{2(1-\nu)\log \rho} = \frac{2\sigma_o}{\sqrt{3}} \tag{7}$$

where T_o and T_ρ are the temperatures at the bore, and $r = \rho$ respectively, E is Young's modulus, and α is the coefficient of linear thermal expansion. This thermal simulation provides an effective method to compute stress intensity factors due to initial stresses given by Eqs. (1) through (4) using NASTRAN or APES. The initial stress is replaced by a temperature input of Eq. (6) at all nodes. The stress intensity factors obtained from NASTRAN or APES corresponding to the thermal loads are equivalent to stress intensity factors due to autofrettage residual stresses.[22]

WEIGHT FUNCTION AND FUNCTIONAL STRESS INTENSITY

A weight function is a universal function which depends only on geometry and not on loadings.[20] If the mode I stress intensity factor $K^{(1)}$ and displacement field $\underset{\sim}{u}^{(1)}$ associated with the symmetric load system 1 are known, the weight function for the cracked geometry is

$$\underset{\sim}{h} = \frac{H}{2K^{(1)}(c)} \frac{\partial \underset{\sim}{u}^{(1)}(c)}{\partial c} \tag{8}$$

where $H = E$ for plane stress and $H = E/(1-\nu^2)$ for plane strain; c is the crack depth. Once $\underset{\sim}{h}$ is determined, the mode I stress intensity factor induced by any other symmetric load system $\underset{\sim}{\tau}$ and $\underset{\sim}{f}$ is given by

$$K = \int_\Gamma (\underset{\sim}{\tau} \cdot \underset{\sim}{h}) d\Gamma + \int_A (\underset{\sim}{f} \cdot \underset{\sim}{h}) dA \tag{8a}$$

where $\underset{\sim}{\tau}$ is the stress vector acting on boundary Γ around the crack tip and $\underset{\sim}{f}$ is the body force in region A defined by Γ. This equation can be reduced to

$$K = \frac{H}{K^{(1)}} \int_o^c p_c(x) \frac{\partial v^{(1)}}{\partial c} dx \tag{9}$$

for radially cracked rings with x being a distance measured along the crack from the base toward the tip. The relation between r and x is

$$r(x) = \begin{cases} 1 + x, & \text{for interior cracks} \\ b - x, & \text{for exterior cracks} \end{cases} \tag{10}$$

The crack pressure $p_c(x)$ can be found from the hoop stress (at the site of radial cracks) in an uncracked ring subjected to the loading of interest. Even though the numerical values of $K^{(1)}$ and $v^{(1)}$, the normal component of displacement, are known, the partial derivative $\partial v^{(1)}/\partial c$ is usually unknown. A technique of computing $\partial v/\partial c$ was devised in Reference 3 by assuming the crack face displacement v be a conic section given by Orange.[23] Another method developed in Reference 15 made no assumptions on v or $\partial v/\partial c$, but utilized the finite element method to compute several stress intensity factors each associated with a simple loading system. For a new load, the new K is expressed in terms of known values of K.

The hoop stress in an uncracked cylinder subjected to an internal pressure p_i is

$$\frac{\sigma_\theta(r)}{p_i} = \frac{1}{b^2-1}\left(1 + \frac{b^2}{r^2}\right) \tag{11}$$

Substituting σ_θ from Eq. (11) as p_c in Eq. (9) we have

$$\frac{K(p_i)}{p_i} = \frac{1}{b^2-1} K_c(1) + \frac{b^2}{b^2-1} K_c(r^{-2}) \tag{12}$$

where

$$K_c(1) = \frac{H}{K(1)} \int_o^c \frac{\partial v(1)}{\partial c}\, dx \tag{13}$$

$$K_c(r^{-2}) = \frac{H}{K(1)} \int_o^c [r(x)]^{-2} \frac{\partial v(1)}{\partial c}\, dx \tag{14}$$

are called functional stress intensity factors.

Similarly we get

$$\frac{K(p_o)}{p_o} = \frac{b^2}{b^2-1} K_c(1) + \frac{b^2}{b^2-1} K_c(r^{-2}) \tag{15}$$

$$\frac{K(\rho=b)}{\sigma_o} = \frac{1}{\sqrt{3}} \{[2 - P_1(b)]K_c(1) - P_1(b)K_c(r^{-2}) + 2K_c(\log r)\} \tag{16}$$

for the same cylinder subjected to uniform tension p_o on outer cylindrical surface and fully autofrettaged residual stress respectively. In Eq. (16) the new functional stress intensity factor is

$$K_c(\log r) = \frac{H}{K(1)} \int_o^c \log(r(x)) \frac{\partial v(1)}{\partial x}\, dx \tag{17}$$

The finite element results of $K(p_i)/p_i$, $K(p_o)/p_o$ and $K(\rho=b)/\sigma_o$ enable us to compute the functional stress intensity factors $K_c(1)$, $K_c(r^{-2})$, and $K_c(\log r)$. For the same flawed cylinder with a different degree of autofrettage, the stressses can be computed from one of the following algebraic equations. For an inner crack with the crack tip r_c in the range $1 \leq r_c \leq \rho$, the equation is

$$\frac{K(\rho)}{\sigma_o} = \frac{1}{\sqrt{3}} \{[2 - P_1(\rho)]K_c(1) - P_1(\rho)K_c(r^{-2}) + 2K_c(\log r)\} \qquad (18)$$

For an outer crack with r_c in the range $\rho \leq r_c \leq b$, the equation is

$$\frac{K(\rho)}{\sigma_o} = \frac{1}{\sqrt{3}} [\rho^2 - P_1(\rho)][b^{-2}K_c(1) + K_c(r^{-2})] \qquad (19)$$

MODIFICATION FORMULAS

For a partially autofrettaged cylinder, let ε be the degree of autofrettage, then $\varepsilon = (\rho-1)/t$ where $t = b - 1$ is the wall thickness of the cylinder. When a crack crosses the elastic-plastic interface $r = \rho$, the hoop stress along the crack face must be represented by both Eqs. (3) and (4). Hence Eqs. (18) and (19) are not valid in such a situation. For inner cracks, we may use Eq. (18) to compute an approximate value which is based on the crack face loading of Eq. (3). The error introduced by Eq. (18) may be corrected by adding the following crack face loading

$$p_c(r) = \begin{cases} 0, & 1 \leq r \leq \rho \\ \dfrac{\sigma_o}{\sqrt{3}} (\rho^2 - P_1)\left(\dfrac{1}{b^2} + \dfrac{1}{r^2}\right) - \dfrac{\sigma_o}{\sqrt{3}} \left\{2 \log \dfrac{r}{\rho} + 1 + \dfrac{\rho^2}{b^2} - P_1\left(\dfrac{1}{b^2} + \dfrac{1}{r^2}\right)\right\}, & \rho \leq r \leq r_c \end{cases} \qquad (20)$$

where r_c is the radius of the crack tip. Substituting from the above into Eq. (9), we should be able to obtain a correction stress intensity factor K_δ if $\partial v/\partial c$ is known. Assume the crack tip crosses the elastic-plastic interface only slightly, the Westergaard near field solution for v in terms of crack-tip stress intensity factor $K^{(1)}$ can be approximately used to find $\partial v/\partial c$. Let ξ be a length measured from the crack tip and be defined by

$$\xi = -(x-c) \tag{21}$$

then

$$v(\xi) = \frac{2K^{(1)}}{H} \left(\frac{2}{\pi}\right)^{1/2} \sqrt{\xi} \tag{22}$$

and

$$\frac{\partial v}{\partial c} = \frac{K^{(1)}}{H} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{1}{\sqrt{\xi}} + \frac{1}{c} \sqrt{\xi}\right) \tag{23}$$

The approximate correction factor K_δ obtained by termwise integration of Eq. (9) using Eqs. (20) and (23) is given by

$$\frac{K_\delta}{\sigma_o} = \frac{1}{\sqrt{3}} \sqrt{2/\pi} \{-1 + 2 \log \rho)(I_1 + I_1') + \rho^2(I_2 + I_2') - 2(I_3 + I_3')\} \tag{24}$$

Using $\delta t = |r_c - \rho|$, the abbreviations in Eq. (24) are:

$$I_1 = 2\sqrt{\delta t} \quad , \quad I_1' = \frac{2}{3c} (\delta t)^{3/2} \tag{25}$$

$$\begin{aligned} I_2 &= \frac{1}{1+c} \left[\frac{\sqrt{\delta t}}{\rho} - \frac{(1+c)^{-1/2}}{2} \log D(\rho)\right] \\ I_2' &= \frac{1}{c} \left[\frac{\sqrt{\delta t}}{\rho} + \frac{(1+c)^{-1/2}}{2} \log D(\rho)\right] \end{aligned} \tag{26}$$

$$I_3 = -2[(2-\log\rho)\sqrt{\delta t} + (1+c)^{1/2}\log\frac{\sqrt{1+c}-\sqrt{\delta t}}{\sqrt{1+c}+\sqrt{\delta t}}] \tag{27}$$

$$I_3' = \frac{2}{3c}[\sqrt{\delta t}\{\delta t\log\rho - 2(1+c) - 2\delta t/3\} - (1+c)^{3/2}\log D(\rho)]$$

with

$$D(\rho) = [2(1+c) - \rho - 2\sqrt{\delta t(1+c)}]/\rho \tag{28}$$

The sum of K_δ/σ_o from Eq. (24) and $K(\rho)/\sigma_o$ from Eq. (18) usually gives a better approximation of crack-tip stress intensity factors when $\delta = |r_c-\rho|/t$ is small. Since the correction formula K_δ/σ_o is not a function of N, the number of cracks, it works for small N and δ. But when N is large, the crack interaction is strong, and δ must be small.

A similar formula can be found for exterior cracks crossing the elastic-plastic interface from elastic into plastic region.

$$\frac{K_\delta}{\sigma_o} = (\frac{2}{3\pi})^{1/2}\{(1-2\log\rho)(J_1+J_1') - \rho^2(J_2+J_2') + 2(J_3+J_3')\} \tag{29}$$

where

$$J_1 = 2\sqrt{\delta t}\quad,\quad J_1' = \frac{2}{3c}(\delta t)^{3/2} \tag{30}$$

$$J_2 = \frac{1}{r_c}[\frac{\sqrt{\delta t}}{\rho} + r_c^{-1/2}\tan^{-1}\sqrt{\delta t/r_c}] \tag{31}$$

$$J_2' = \frac{1}{c}[-\frac{\sqrt{\delta t}}{\rho} + r_c^{-1/2}\tan^{-1}\sqrt{\delta t/r_c}] \tag{32}$$

$$J_3 = 2[(-2+\log\rho)\sqrt{\delta t} + 2\sqrt{r_c}\tan^{-1}\sqrt{\delta t/r_c}] \tag{33}$$

$$J_3' = \frac{2}{3c}[(\delta t)^{3/2}\log\rho + \frac{2}{3}(3r_c-\delta t)\sqrt{\delta t} - 2r_c^{3/2}\tan^{-1}\sqrt{\delta t/r_c}] \tag{34}$$

Adding K_δ/σ_o of Eq. (29) to $K(\rho)/\sigma_o$ of Eq. (19), the result is the corrected crack-tip stress intensity factors for exterior cracks.

NUMERICAL RESULTS AND CONCLUSIONS

Extensive numerical results are obtained for a cylinder of b = 2, which is a commonly used value in cannon design. A typical finite element idealization is shown in Figure 1 for inner radial cracks. Slight modifications in element meshes are needed for exterior cracks. In Figure 1 the elements surrounding the crack tip are enriched elements.[10] If collapsed singular elements are to be used, we can simply replace the enriched quadrilaterals by collapsed quadrilateral elements (triangular elements) with proper shifting of side nodes to new locations. Similar accuracy is achieved using either enriched or collapsed quadrilaterals. Stress intensity factors for internal cracks are given in Table 1 of Reference 15 for three different types of loadings. Using these results and using Eqs. (12), (15), and (16), we obtain functional stress intensity factors for internal cracks. Figure 2 shows stress intensity factors as a function of c/t for various numbers of internal cracks in a fully autofrettaged cylinder. Figures 3, 4, and 5 are similar graphs of functional stress intensity factors. Corresponding graphs for external cracks are shown in Figures 6 through 9.

Readings taken from Figures 3, 4, and 5 are enough for an estimate of stress intensity factor for internally cracked cylinders with any assigned values of N and c/t for any combination of p_i and residual stresses corresponding to a given ϵ. If crack tips cross the elastic-plastic

interface, then correction formula (24) should be used. As an example, if the stress intensity factor is desired for N = 2, c/t = 0.3 in a 25 percent autofrettaged cylinder, we first take readings: $K_c(1)/\sqrt{\pi c} = 1.41$ from Figure 3, $K_c(r^{-2})/\sqrt{\pi c} = 1.05$ from Figure 4, and $K_c(\log r)/\sqrt{\pi c} = 0.22$ from Figure 5; then $K_c(\varepsilon=0.25)/\sigma_o\sqrt{\pi c} = -0.12$ is computed from Eq. (18). Since $r_c = 1.3$ is greater than $\rho = 1.25$, the correction stress intensity factor $K_\delta/\sigma_o\sqrt{\pi c} = -0.023$ is obtained from Eq. (24). The sum of Eqs. (18) and (24) gives $K/\sigma_o\sqrt{\pi c} = -0.143$. To check this result, a finite element computation of this case is performed. The result is also -0.143. For externally cracked cylinders, Eq. (19) involves only two functional stress intensity factors. Therefore, only Figures 7 and 8 are needed. For example, given N = 4, readings taken from Figure 7 are $K_c(1)/\sqrt{\pi c} = 1.12$ and 1.18 for c/t = 0.2 and 0.3 respectively. Readings are $K_c(r^{-2})/\sqrt{\pi c} = 0.32$ for c/t = 0.2 and 0.36 for c/t = 0.3 from Figure 8. Stress intensity factors for $\varepsilon = 0.8$ can be computed from Eq. (19), giving $K(\varepsilon=0.8)/\sigma_o\sqrt{\pi c} = 0.492$ and 0.537 for c/t = 0.2 and 0.3 respectively. For c/t = 0.3, the correction formula (29) must be used. The result is $K_\delta/\sigma_o\sqrt{\pi c} = -0.05$. The final result for c/t = 0.3 is $K(\varepsilon=0.8)/\sigma_o = 0.487$ which is close to the result of 0.486 obtained directly from a finite element computation.

For a combination of residual stresses and internal pressure, the stress intensity factor is simply an algebraic sum. Stress intensity factors normalized by $\sigma_o\sqrt{\pi c}$ are shown in Figure 10 as a function of N for internal cracks subjected to several selected values of p_i and ε. Figure 11 is a similar graph for external cracks.

The functional stress intensity factors are used to obviate the difficulty in finding the weight function itself. The approach is the result of a series of methods developed for multiply cracked cylinders. The extension of the method to residual stress distribution other than that given by Eqs. (1) through (4) is highly possible.

From numerical results, the stress intensity factor is largest for N = 2 for various combinations of residual stresses and internal pressures for both interior and exterior cracks. The stress intensity factor monotonically decreases as the number of cracks increases from N = 2.

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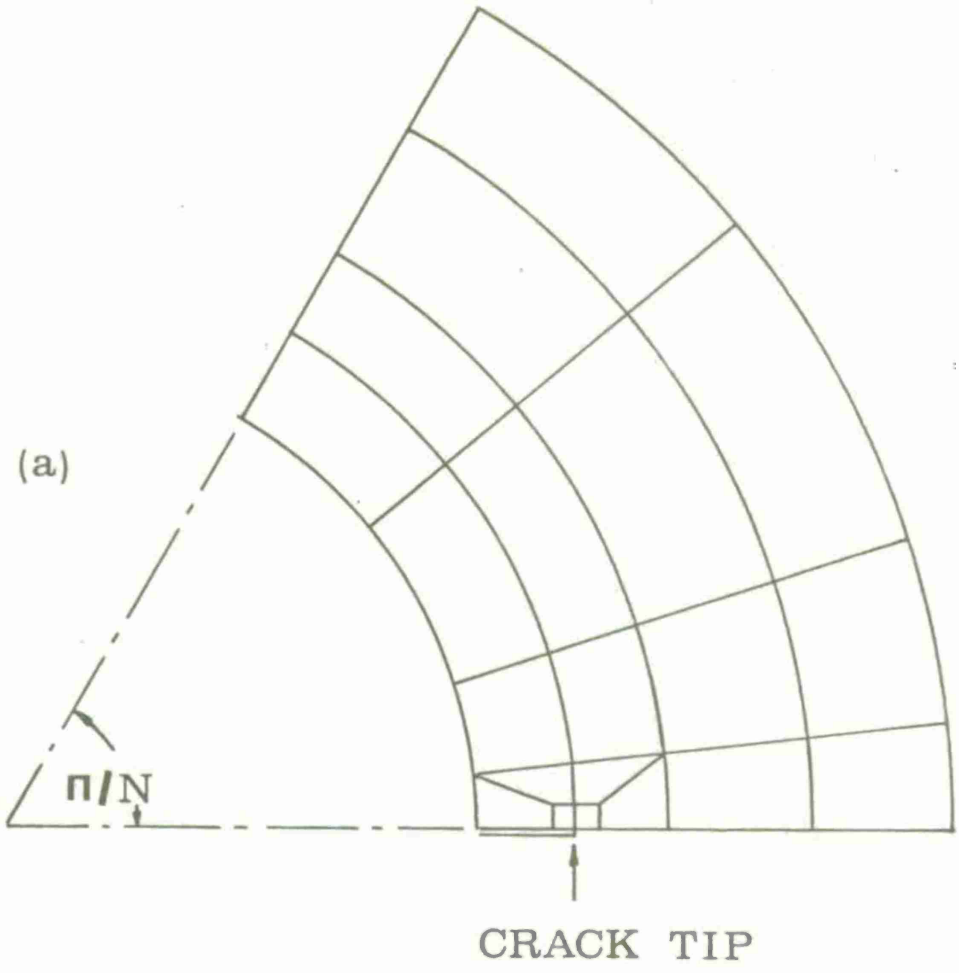

Figure 1(a). A typical finite element idealization.

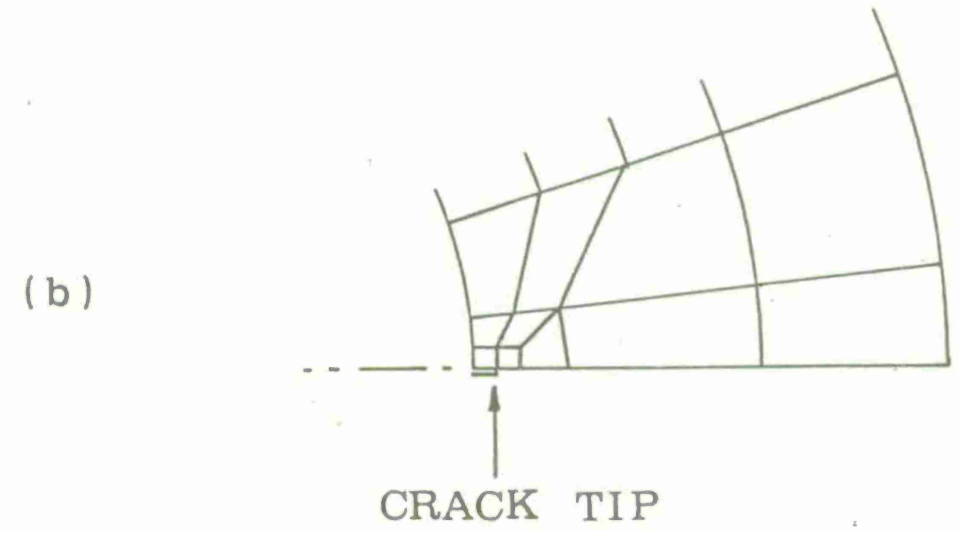

Figure 1(b). Idealization for very shallow cracks.

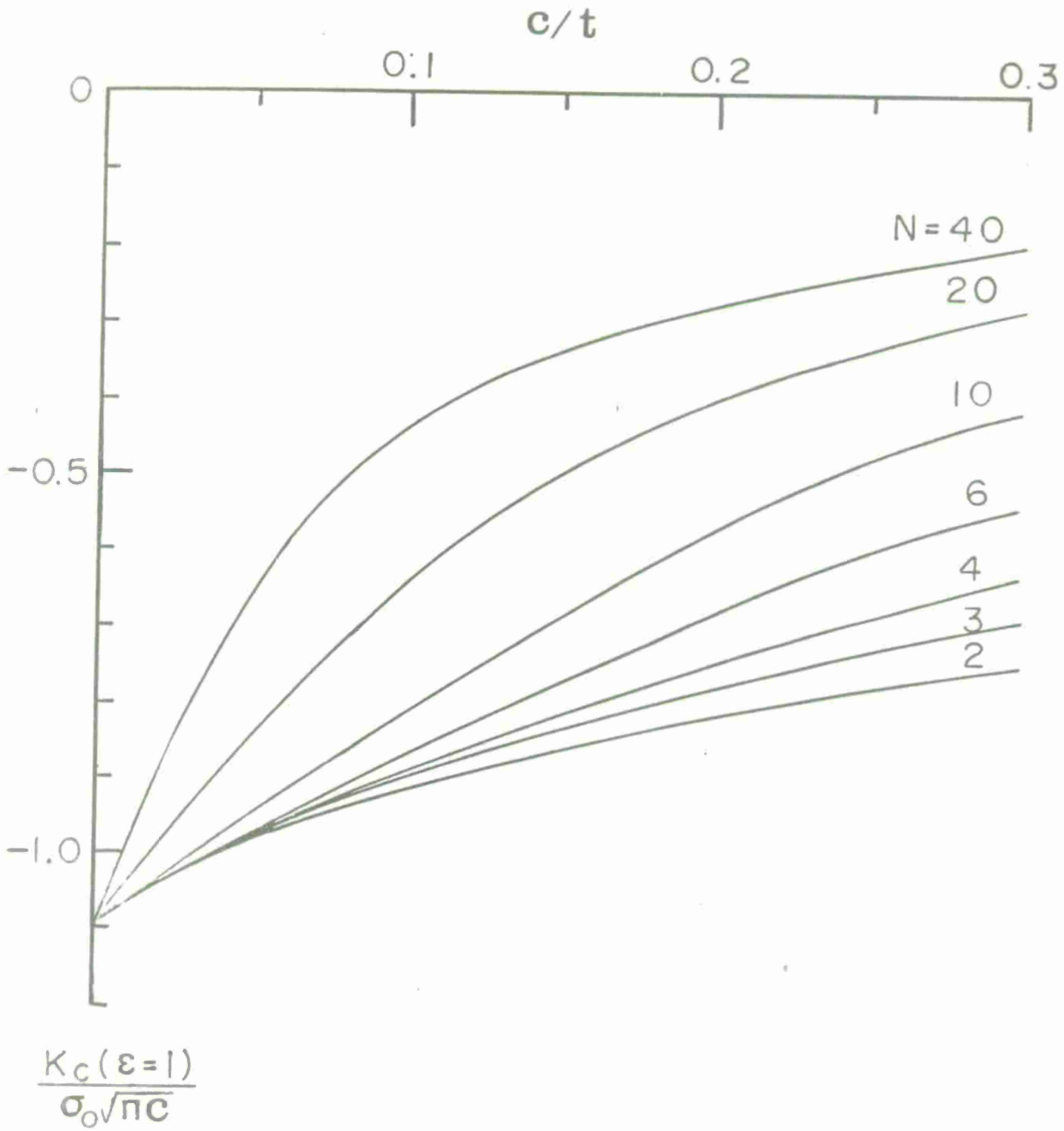

Figure 2. Stress intensity factor as a function of c/t for N ID cracks in a fully autofrettaged cylinder of b/a = 2.

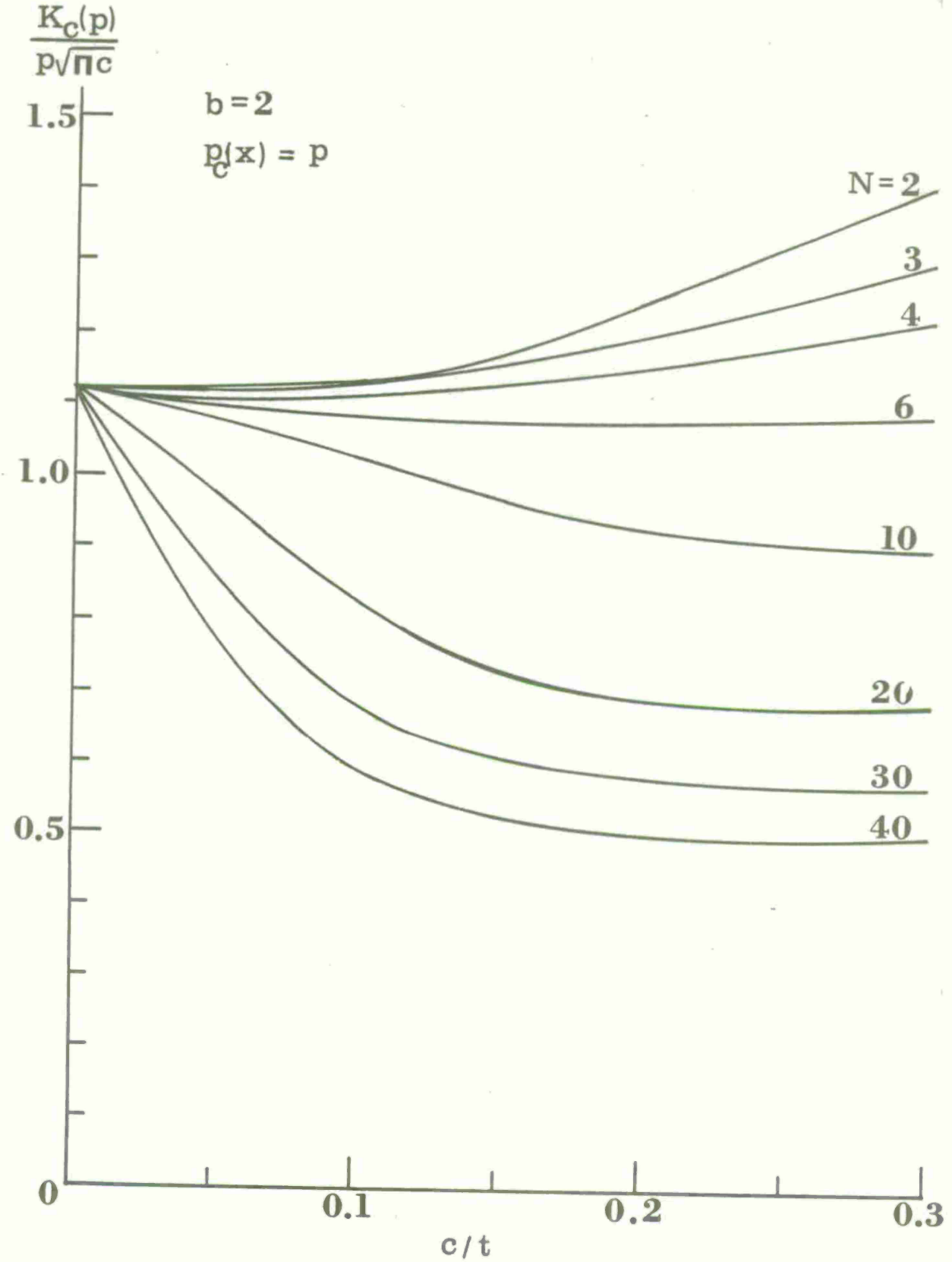

Figure 3. $K_c/p\sqrt{\pi c}$ as a function of c/t for N internal radial cracks with constant crack face loading.

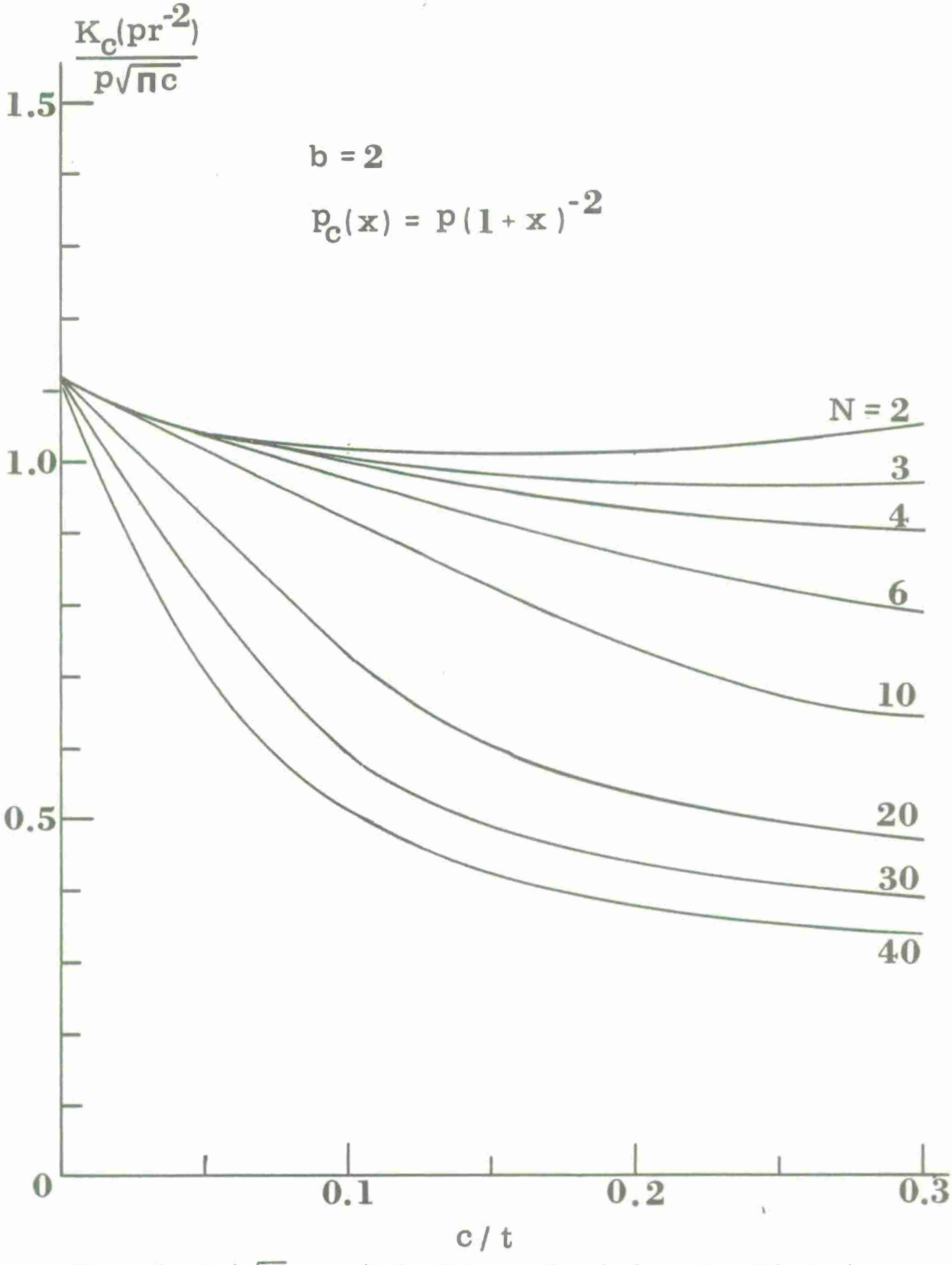

Figure 4. $K_c/p\sqrt{\pi c}$ vs. c/t for N internal radial cracks with crack face loading $p_c(x) = p(1+x)^2$.

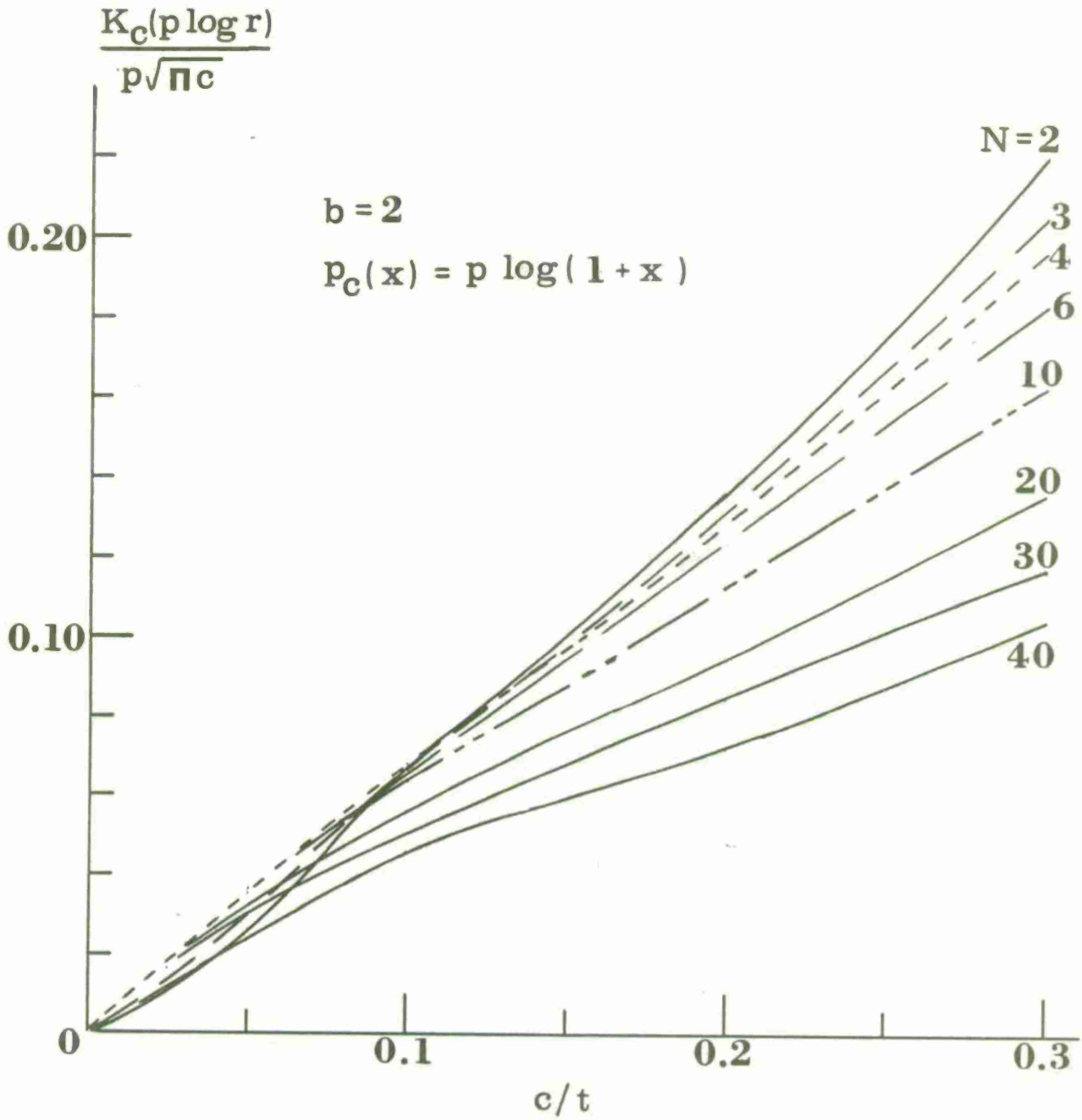

Figure 5. $K_c/p\sqrt{\pi c}$ vs. c/t for N internal radial cracks with crack face loading $p_c(x) = p \log(1+x)$.

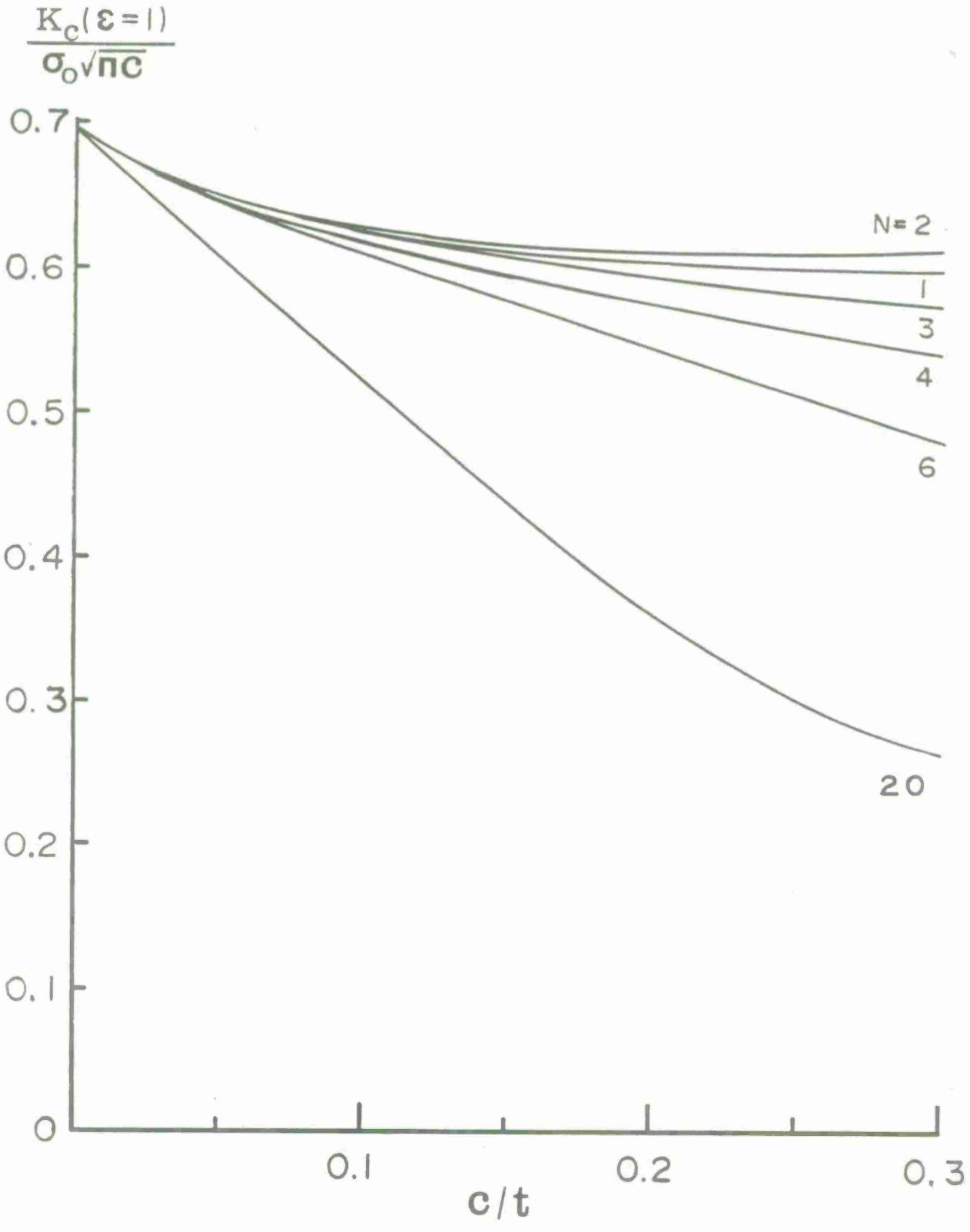

Figure 6. $K/\sigma_o\sqrt{\pi c}$ as a function of c/t for N radial cracks at outer surface of a fully autofrettaged cylinder of b/a = 2.

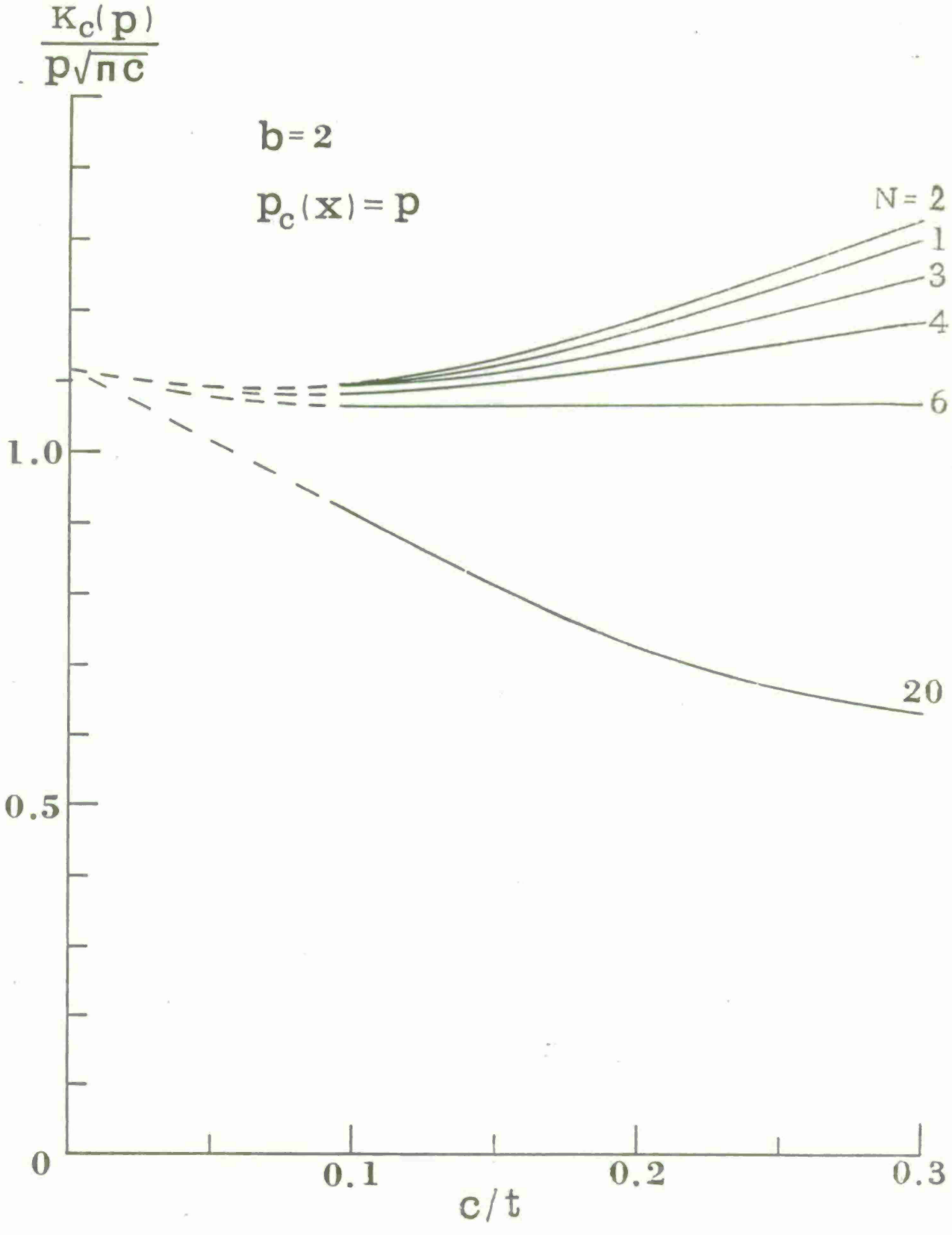

Figure 7. $K_c(p)/p\sqrt{\pi c}$ as a function of c/t for N external radial cracks with constant crack face loading.

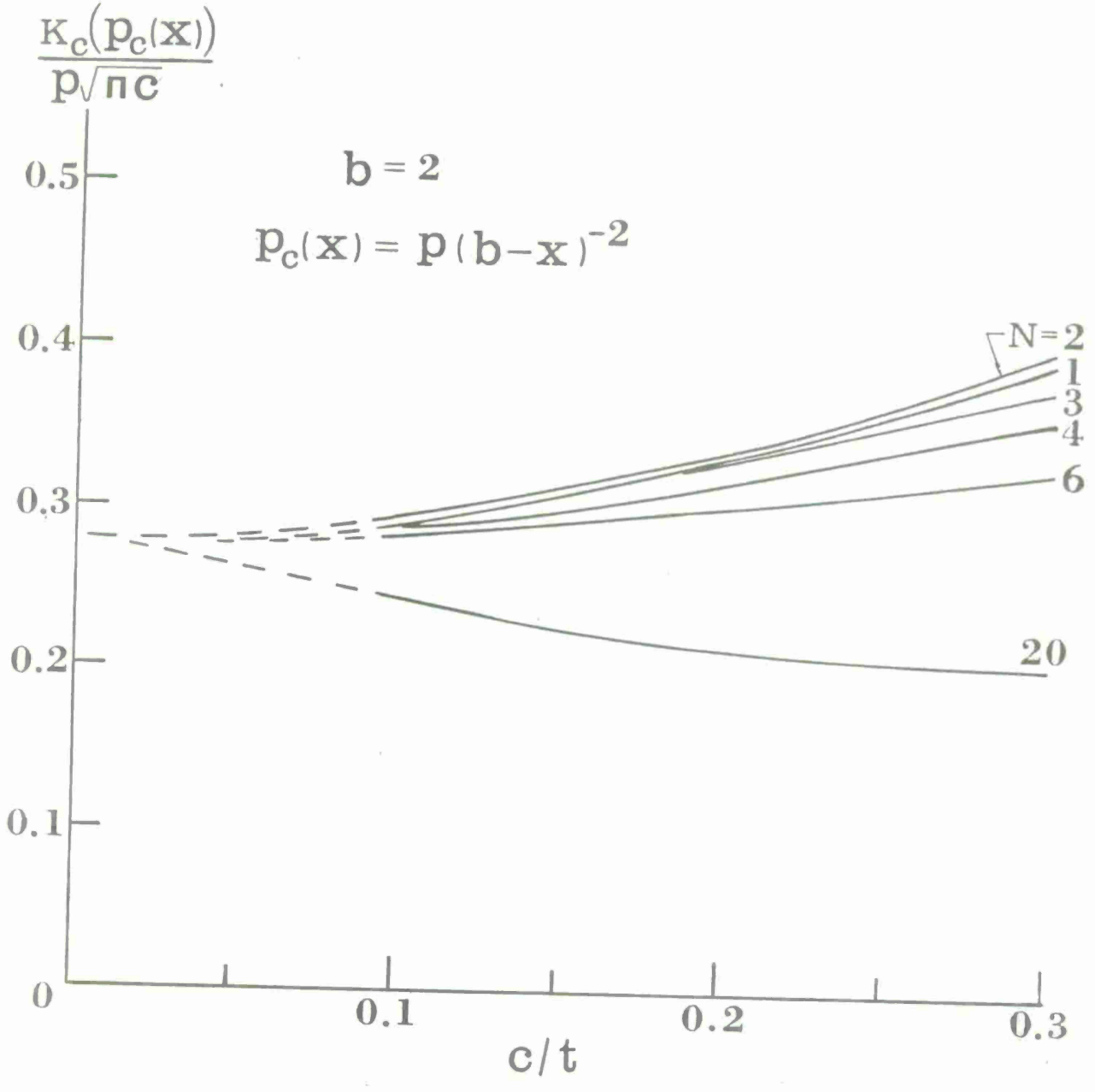

Figure 8. $K_c/p\sqrt{\pi c}$ vs. c/t for N external radial cracks with crack face loading $p_c(x) = p(b-x)^{-2}$.

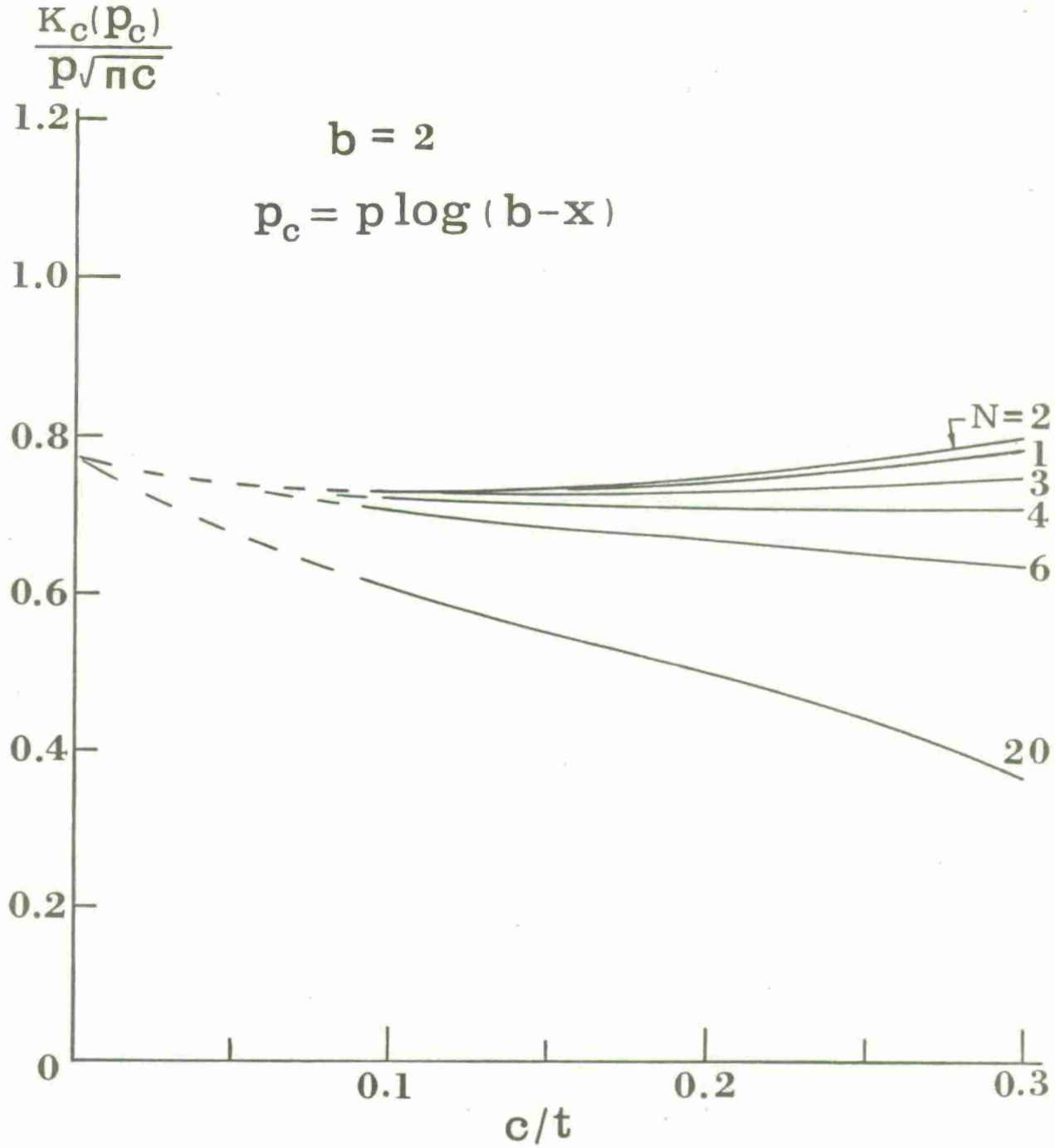

Figure 9. $K_c/p\sqrt{\pi c}$ vs. c/t for N external radial cracks with crack face loading $p_c(x) = p \log(b-x)$.

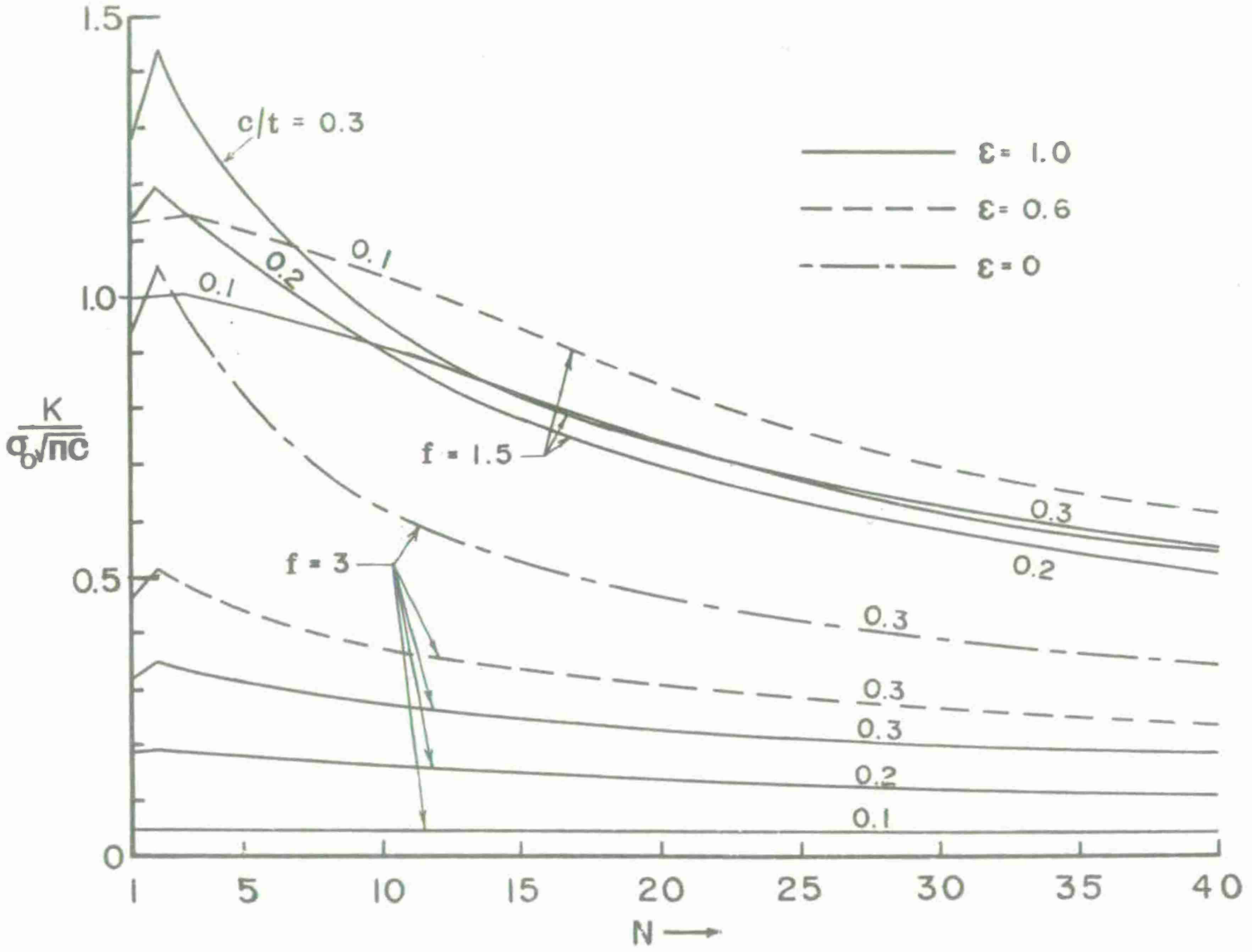

Figure 10. $K/\sigma_o\sqrt{\pi c}$ for N radial cracks at inner surface of a cylinder of b/a = 2 subjected to combined internal pressure $p_i = \sigma_o/f$ and residual stresses corresponding to given degrees of autofrettage, ε.

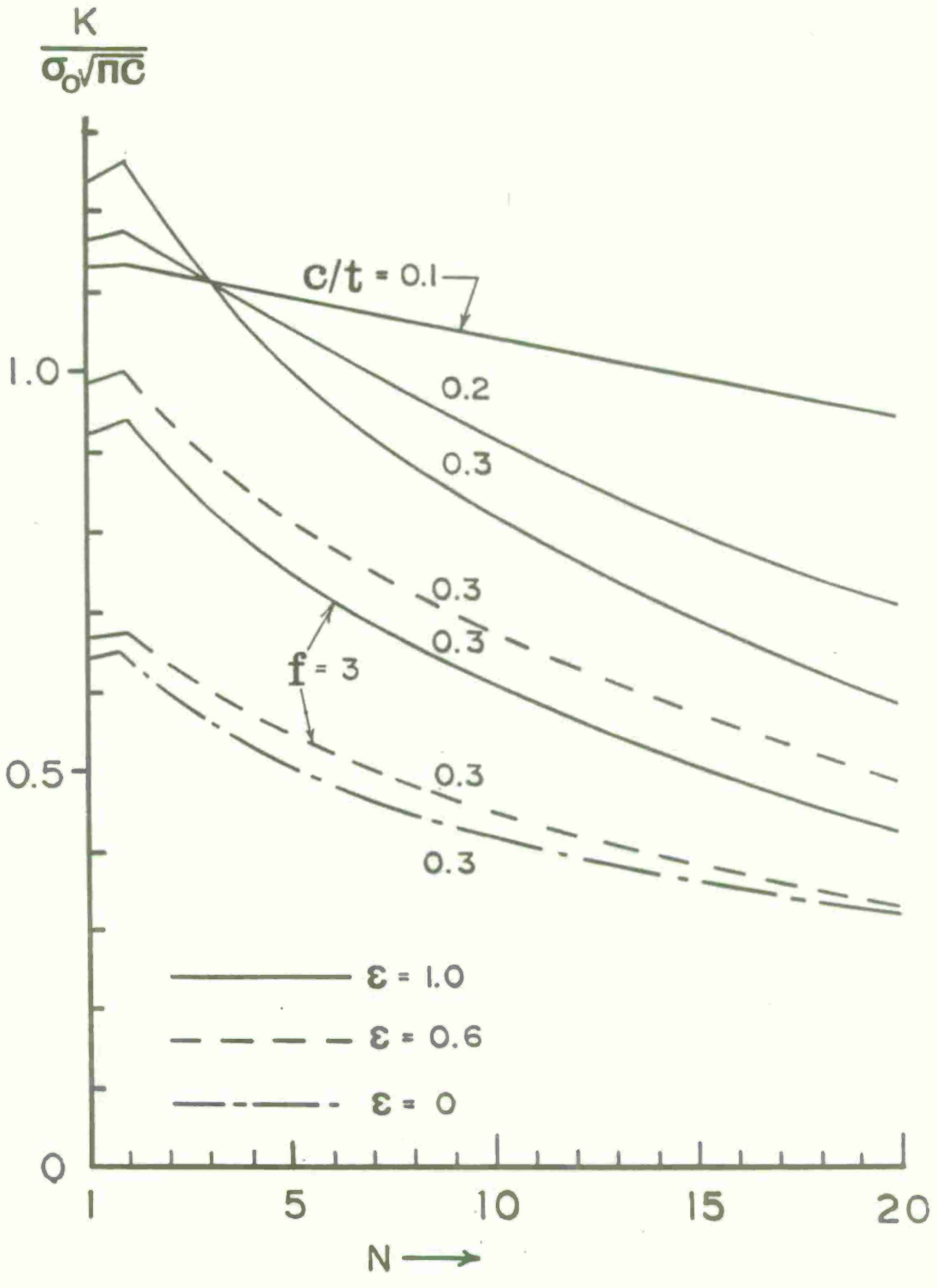

Figure 11. $K/\sigma_o\sqrt{\pi c}$ for N radial cracks at outer surface of a cylinder of $b/a = 2$ subjected to combined internal pressure $p_i = \sigma_o/f$, where $f = 1.5$ except otherwise indicated, and residual stresses corresponding to given degrees of autofrettage ε.

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